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Traffic Flow Modelling in Teaching Differential Equations

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Traffic Flow Modelling in Teaching Differential Equations

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This talk explores the pedagogical strategy of employing traffic flow models to enrich the teaching and understanding of differential equations. It begins by establishing the significance of modelling real-world phenomena, specifically traffic flow, to demonstrate the practical applications of differential equations.

Significance of modelling of real-world problems

- Why use real-world models?
- Demonstrating practical applications of differential equations
  - Traffic Flow

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Why? Incorporating real-world models enriches the teaching of differential equations by making the subject more relevant, engaging, and beneficial to students.

Real-world modeling enriches the educational experience and equips students with valuable skills and perspectives.

Differential equations have a wide range of practical applications across numerous fields, demonstrating their significance in understanding and solving real-world problems. Differential equations are essential tools for understanding and predicting various real-world phenomena.

Traffic Flow: As discussed in this presentation, they are used to model traffic flow, optimizing transportation systems and minimizing congestion.

Unit Learning Outcomes

	On successful completion of this section students can	Graduate Capabilities addressed
1	Appy traffic laws and theories to govern traffic flow and the corresponding mathematical models	
2	Use empirical or theoretical functions to represent the fundamental diagram	
3	Apply proper mathematical approaches to solve mathematical model of traffic flow..	

On successfully completing all of the assessments you will have achieved all of these learning outcomes.

1.Apply discipline knowledge, principles and concepts

2.Innovative, creative and risk-taking, and resourcefulness to create, develop, and manage a business or venture.

3. Effective communicators with digital competency

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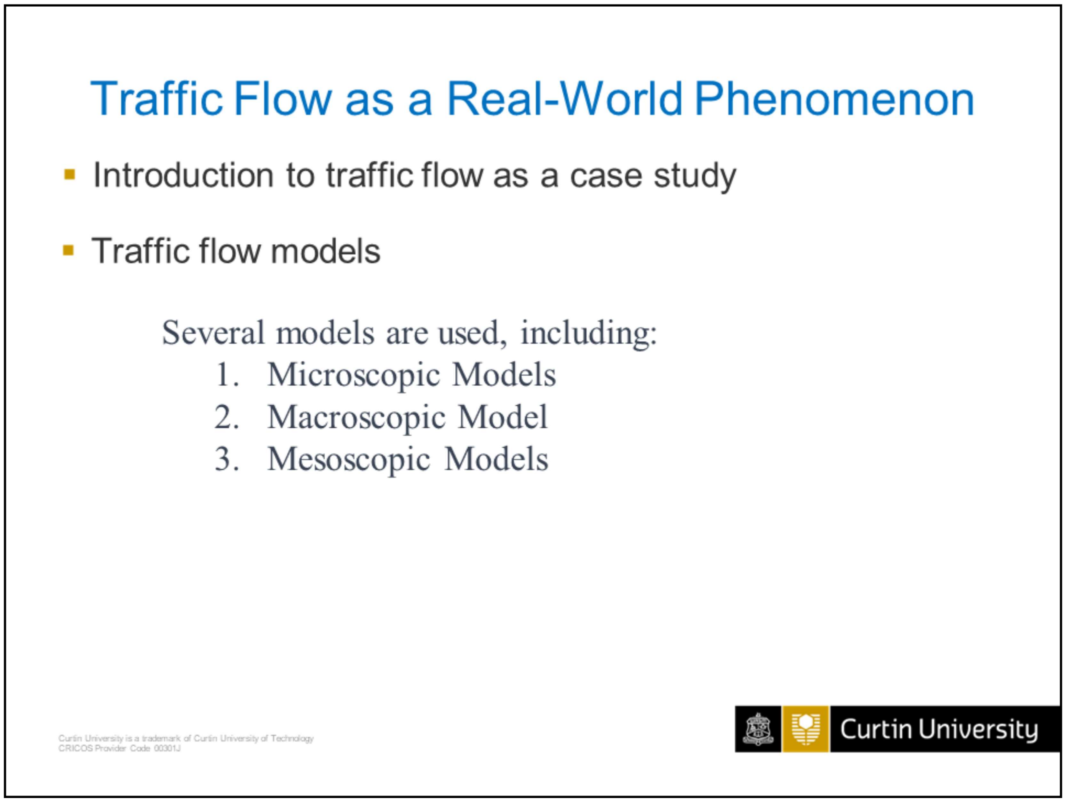


Traffic flow serves as an insightful case study to explore the application of differential equations in modeling and improving real-world systems.

Traffic flow refers to the movement of vehicles on road networks and the dynamics of how vehicles interact with one another.

Traffic congestion is a common problem in urban areas worldwide, making traffic flow an ideal case study.

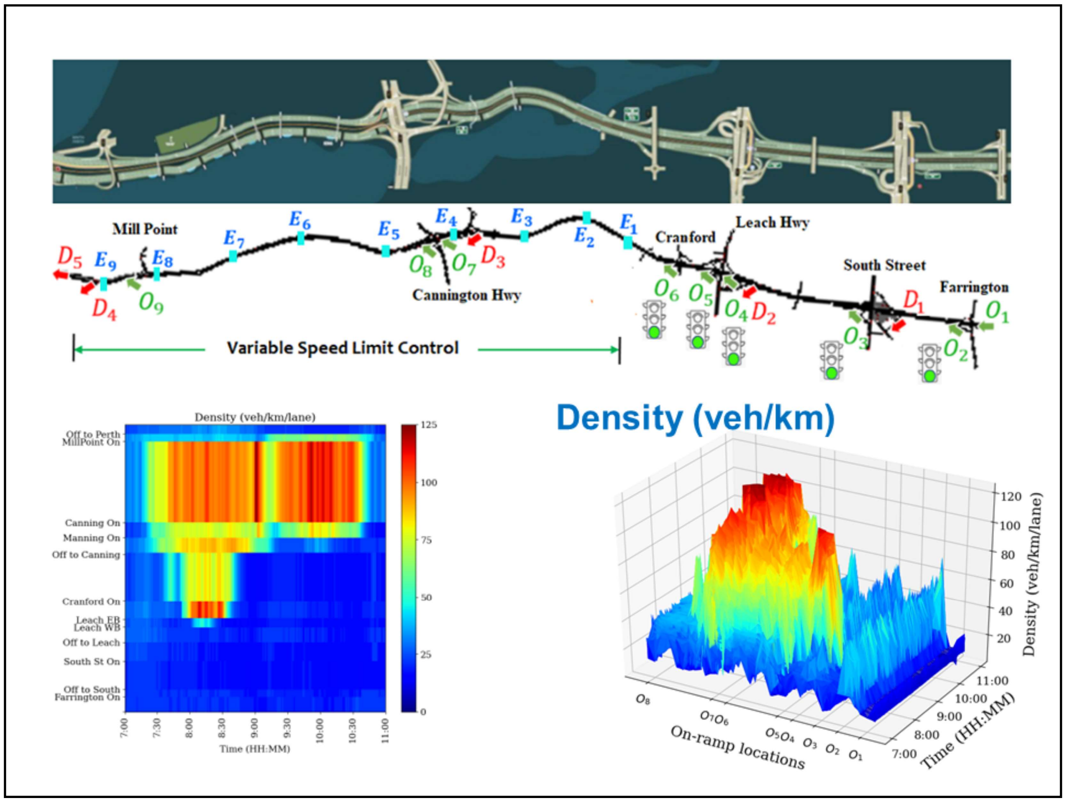
Traffic flow modeling encompasses various mathematical approaches to understand and manage traffic behavior, making it a valuable application of differential equations.

1.Traffic Flow Models: Traffic flow modeling involves creating mathematical representations of how vehicles move on road networks.

2.Purpose: These models help predict, analyze, and optimize traffic behavior and congestion patterns.

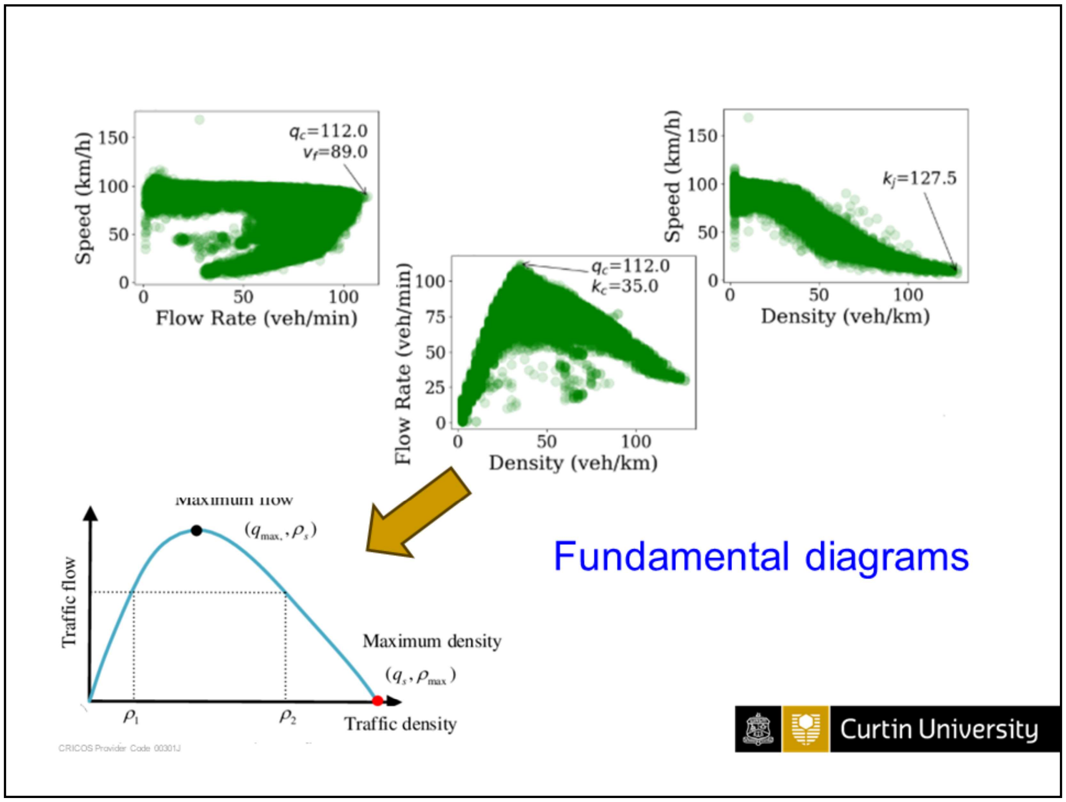
3.Types of Models: Several models are used, including:

- **Microscopic Models**: Focus on individual vehicle behavior and interactions.
- **Macroscopic Models**: Describe traffic flow in terms of density, speed, and flow at a larger scale.
- **Mesoscopic Models**: Combine elements of both microscopic and macroscopic models.



Understanding the mathematical relationships and using differential equations is essential in modeling traffic flow and addressing traffic-related challenges.

Traffic flow models are based on fundamental mathematical relationships between flow (F), density (ρ), and velocity (v).

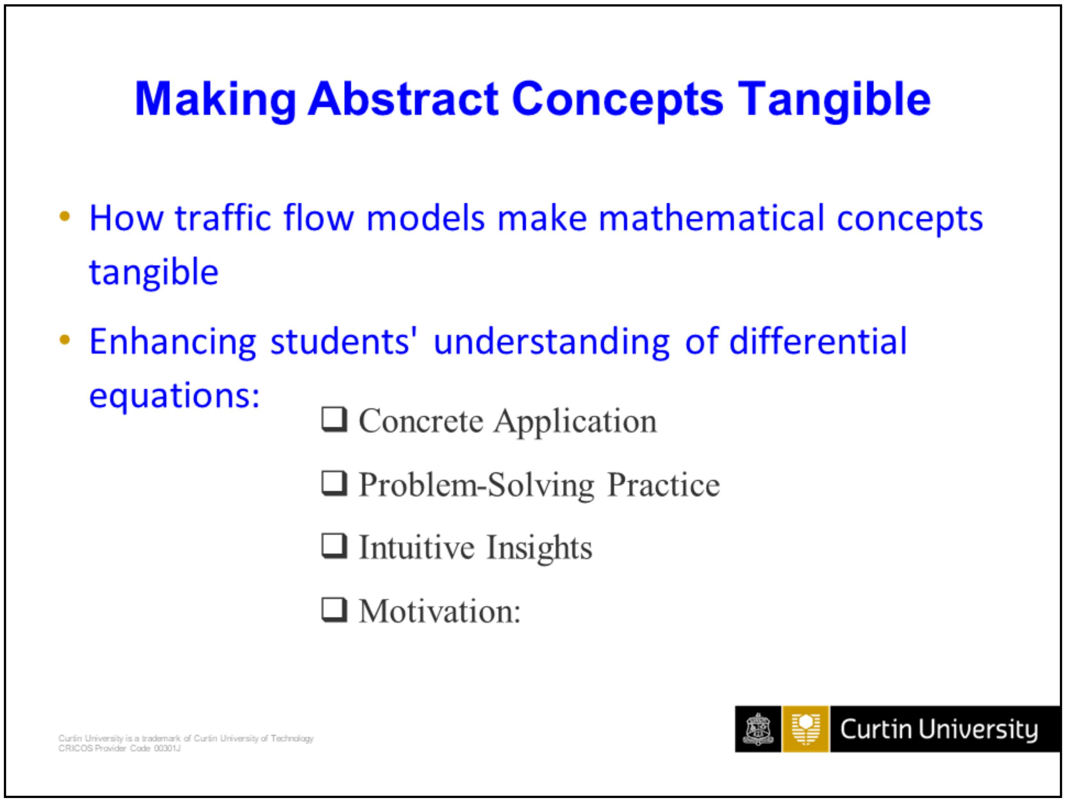


1.Fundamental Equation: The fundamental equation in traffic flow modeling is:

1. Flow (F) = Density (ρ) × Velocity (v)

2.Inverse Relationship: In congested traffic (high density), velocity tends to decrease, resulting in a lower flow. Conversely, in lighter traffic (low density), vehicles can move faster, leading to higher flow.

3.Differential Equations in Traffic Flow: Traffic flow models often involve differential equations to describe how density, velocity, and flow change over time and space.



Students can relate to the familiar experience of traffic congestion and movement. Traffic models show how differential equations capture dynamic interactions between vehicles, speed changes, and traffic patterns. Interactive simulations allow students to experiment with different scenarios, observing how changes in variables impact traffic flow.

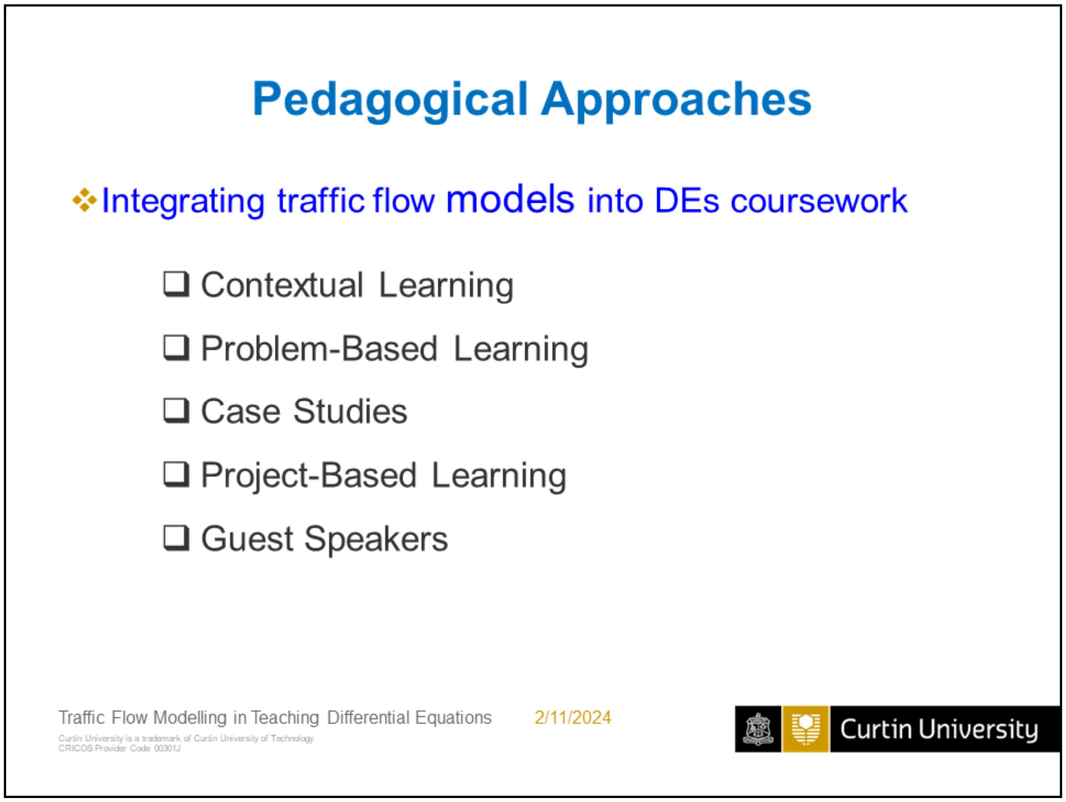
Enhancing Students' Understanding of Differential Equations:

1.**Concrete Application**: Traffic flow models illustrate how differential equations are used to solve practical problems, making the application of abstract concepts more apparent.

2.**Problem-Solving Practice**: Students can practice solving differential equations within the context of traffic flow, developing problem-solving skills they can apply elsewhere.

3.**Intuitive Insights**: The tangible nature of traffic flow models helps students intuitively grasp concepts like rates of change, equilibrium, and stability.

4.**Motivation**: The relevance of traffic flow to their daily lives can motivate students to engage more deeply with the mathematical concepts involved.



Integrating traffic flow models and interactive simulations into differential equations coursework enhances learning by providing practical experiences and promoting active engagement.

1.Integrating Traffic Flow Models into Differential Equations Coursework:

1. **Contextual Learning**: Incorporate traffic flow models into the curriculum to provide real-world context for learning differential equations.
2. **Problem-Based Learning**: Present traffic-related problems that require students to apply differential equations to solve them.
3. **Case Studies**: Include real traffic data and scenarios as case studies, allowing students to analyze and model traffic flow.
4. **Project-Based Learning**: Assign projects where students create their traffic flow models, applying differential equations to real or hypothetical traffic situations.
5. **Guest Speakers**: Invite experts or professionals from transportation engineering or urban planning to share insights on the practical use of traffic flow models.

❖Interactive simulations as a teaching tool

- Engagement
- Visualization
- Experimentation
- Data Analysis
- Feedback
- Online Resources

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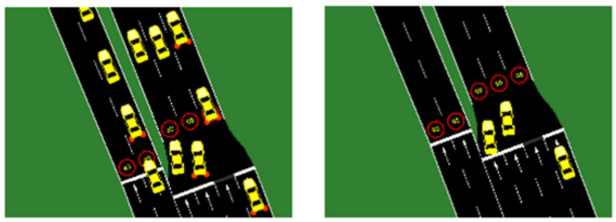
1. **Engagement**: Use interactive simulations to engage students actively in exploring traffic flow concepts.
2. **Visualization**: Simulations provide visual representations of traffic behavior, making complex concepts more accessible.
3. **Experimentation**: Students can experiment with different parameters and scenarios to observe how they impact traffic flow.
4. **Data Analysis**: Simulations generate data that can be analyzed, allowing students to validate their models and hypotheses.
5. **Feedback**: Immediate feedback from simulations helps students refine their understanding and problem-solving skills.
6. **Online Resources**: Utilize online platforms and software that offer interactive traffic flow simulations and modeling tools.

Experiments

- Our microscopic traffic simulation is 24 hours with 86400 incremental time steps of step size $\triangle t = 1$ second.
- All model parameters are determined through the calibration process.
- The linked ramp-metering scheme based on the ALINEA and HERO algorithms were applied at five on-ramp roads on Farrington road, South Street, Leach Highway (2 TLS) and Cranford Avenue.



(a) rush hours (b) non rush hours

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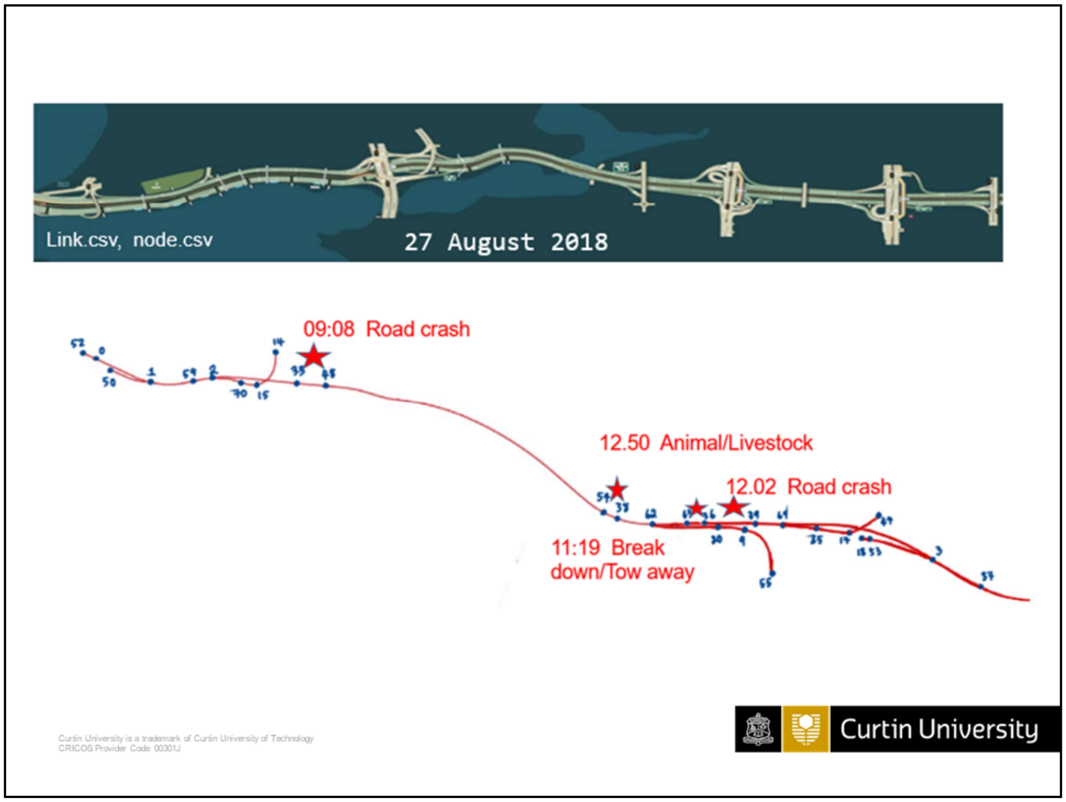
These case studies illustrate how traffic flow models, driven by differential equations, address real-world traffic problems and contribute to more efficient and sustainable transportation systems.

Case Study 1: Highway Congestion Management:

1. Use of traffic flow models to predict and alleviate congestion on a busy highway.
2. Application of differential equations to optimize traffic signal timings, lane management, and congestion mitigation strategies.

Case Study 4: Traffic Flow Prediction:

1. Forecasting traffic conditions using differential equations and historical data.
2. Highlighting the role of mathematical modeling in providing real-time traffic information to commuters.



On 27 August 2018, there were 4 road incidents occurred in the morning, 2 incidents (Road crash at 908 am and Breakdown at 11:19 am) between Leach Hwy on-ramp and Mills Point RD, and other 2 incidents (Road crash at 12:02 pm and Animal/livestock at 12:50 pm) between Manning RD on-ramp and Leach Hwy on-ramp.

Objective function: $f = \min(TTS + \gamma D)$

$$TTS = \sum_{t=1}^{t_0+hT} T\left\{\sum_{i=1}^{m}(L_i k_i(t)) + \sum_{i=1}^{n} N_i(t)\right\}; \qquad D = \sum_{t=1}^{\tau} T\left\{\sum_{i=1}^{rs} L_i\left(k_{i,t} - \frac{L_i q_{i,t}}{(1-\beta)v_f}\right)\right\}$$

Decision variables: $X \in R^{h\times p}$; $p = n_r + n_v$

$$X = \left(\alpha_{1,t}, \alpha_{2,t}, \dots, \alpha_{n_r,t}, \beta_{1,t}, \beta_{2,t}, \dots, \beta_{n_v,t}\right)_{t=1}^{h}, \qquad \begin{cases}\alpha_{min} \le \alpha_{i,t} \le \alpha_{max} \\ \beta_{min} \le \beta_{i,t} \le \beta_{max}\end{cases}$$

$$\alpha_i(t) = \begin{cases}\alpha_{i1} & \text{if } t \in [t_0, t_0 + T_c) \\ \alpha_{i2} & \text{if } t \in [t_0 + T_c, t_0 + 2T_c) \\ \vdots & \\ \alpha_{im} & \text{if } t \in [t_0 + (h-1)T_c, t_0 + hT_c)\end{cases} \qquad (i=1,\dots,n_r)$$

$$\beta_i(t) = \begin{cases}\beta_{i1} & \text{if } t \in [t_0, t_0 + T_c) \\ \beta_{i2} & \text{if } t \in [t_0 + T_c, t_0 + 2T_c) \\ \vdots & \\ \beta_{im} & \text{if } t \in [t_0 + (h-1)T_c, t_0 + hT_c)\end{cases} \qquad (i=1,\dots,n_v)$$

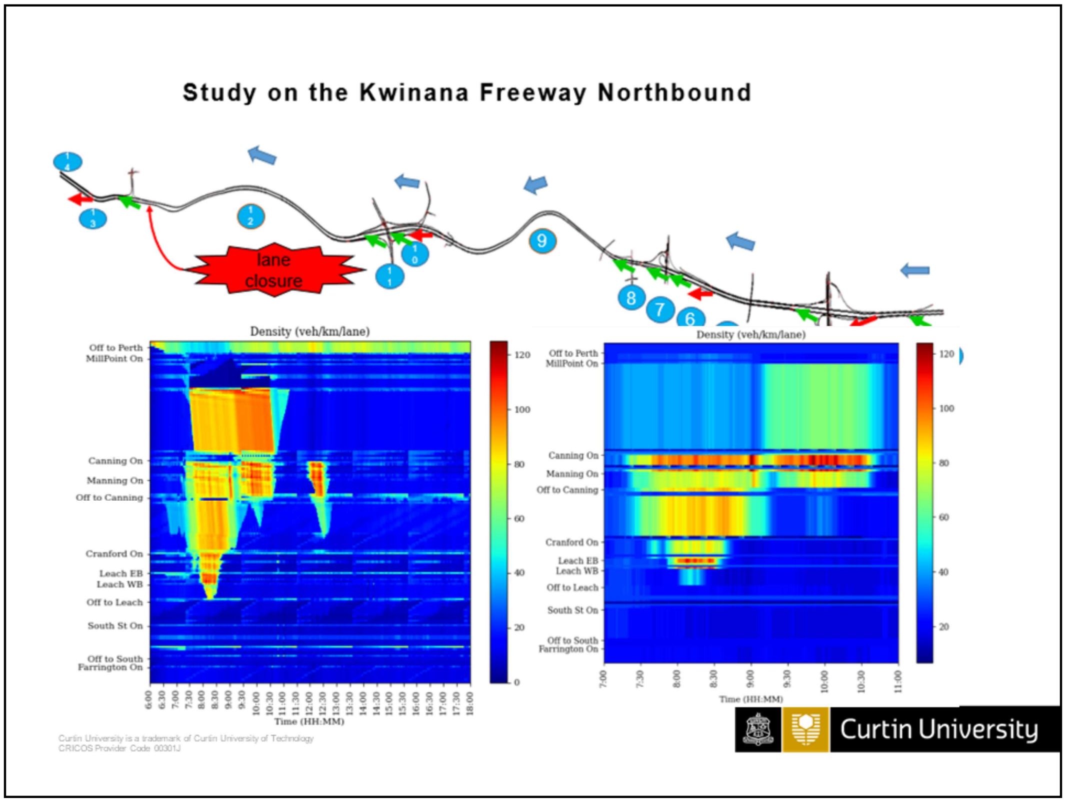
- **Traffic flow constraints**

$$k_{i,t}(\alpha,\beta) = k_{i,t-1}(\alpha,\beta) + \frac{T}{L_i}\left(l_{i-1}q_{i-1,t-1}(\alpha,\beta) - l_i q_{i,t-1}(\alpha,\beta) + r_{i,t-1} - s_{i,t-1}\right)$$

$$N_{i,t} = N_{i,t-1} + T\left(d_{i,t-1} - r_{i,t-1}\right), \qquad 0 \le N_{i,t} \le N_{max}$$

  - $r_{i,t} = (1-\alpha_{i,t})\max\left[r_{i,t}^{LC};\ \hat{r}_{i,t}\right]$
    - $\hat{r}_{i,t} = \left(d_{i,t} + \frac{N_{i,t-1}}{T}\right)$
    - $r_{i,t}^{LC} = r_{i,t-1}^{LC} + \frac{L_i}{T}(k_{max} - k_{i,t-1})$
  - $q_{i,t} = min\{q_{i-1,t};\ q_{max};\ \ (1-\beta_{i,t-1})v_f k_{i,t-1});\ w_{i+1}(t)(k_{max} - k_{i+1,t})\}$

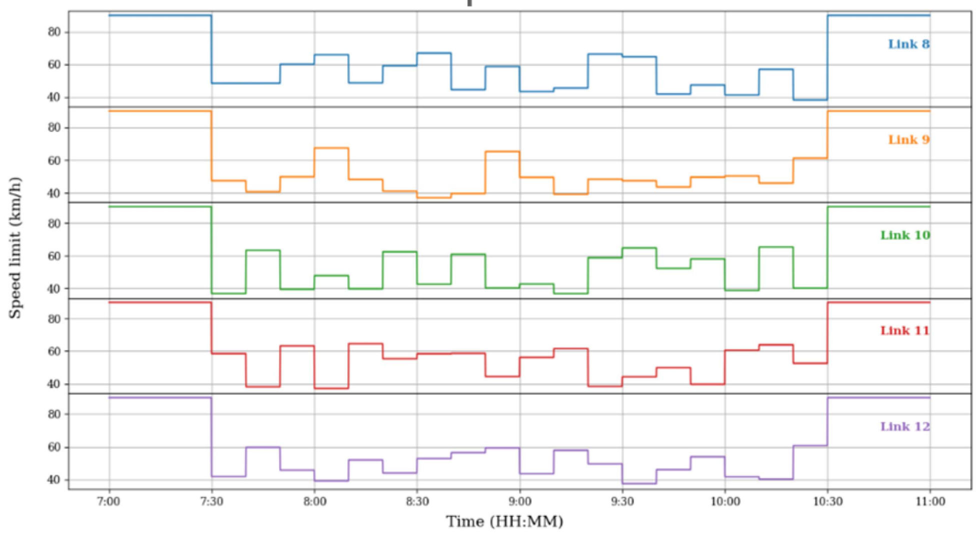
- **Road Capacity constraints**
  - $l_{i-1}q_{i-1,t} + r_{i,t} - s_{i,t} \le q_{max} \quad (i = 1, \ldots, m;\ \ t = 1, \ldots, h)$



Left diagram shows simulated density of traffic obtained from the model with no control

Right diagram is the simulated data obtained from the model with RM & VSL control

VSL profiles



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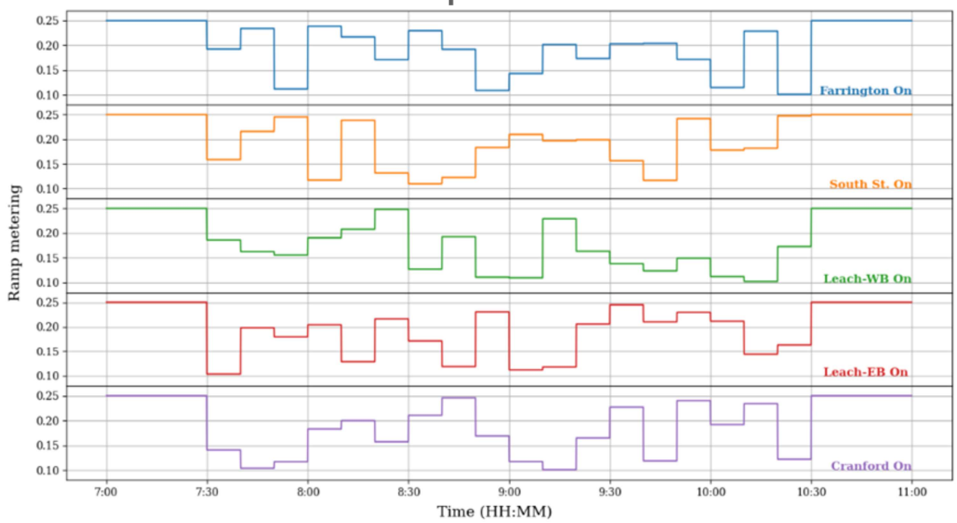
RM profiles



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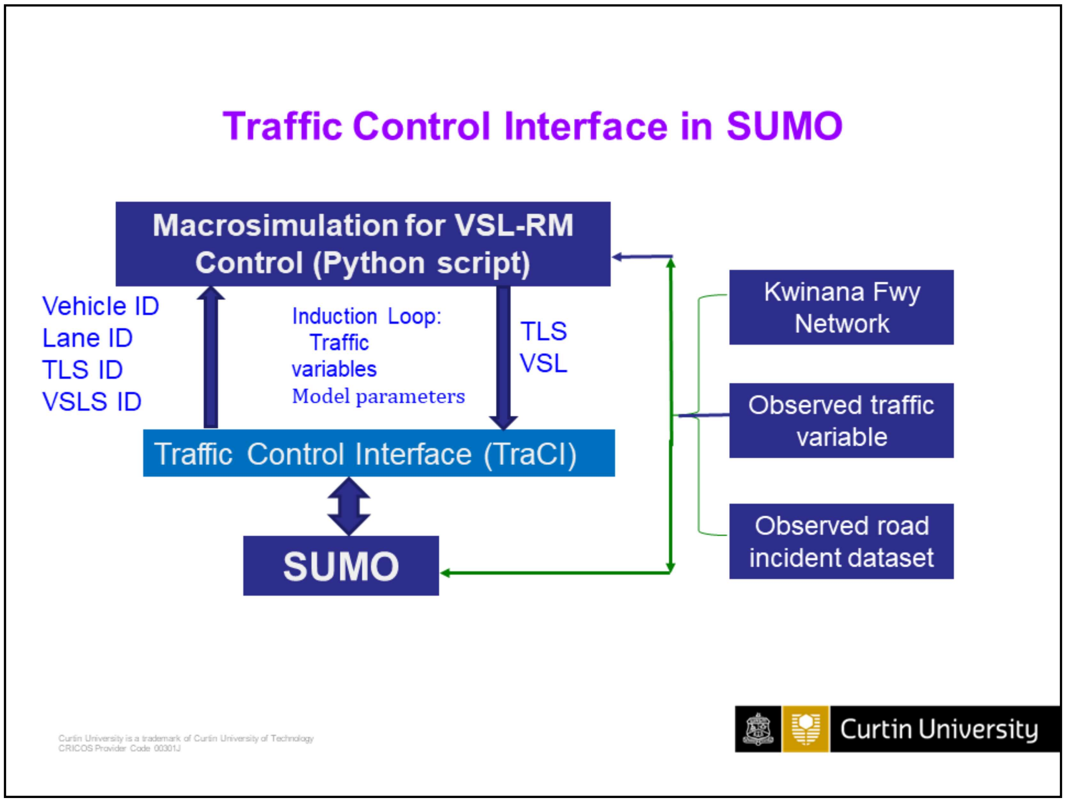

To quantify the safety and **mobility** impact, the proposed integrated VSL-RM control was implemented in microscopic simulation and compared with **VSL** control, **RM control** and without any control scenario respectively. SUMO is used to model the RM and the cooperative RM, the VSLS and the cooperative VSLS.

Prediction - Baseline model

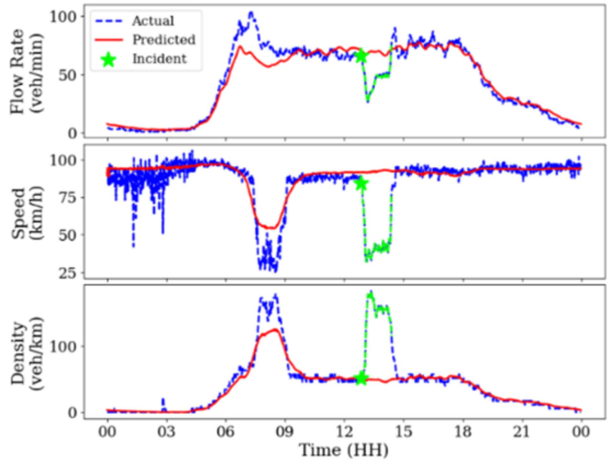

Figure: Baseline predictions of traffic flow rate (veh/min/lane), speed (km/hr) and density (veh/km/lane) with a road incident (∗) and its impact on traffic parameters (a solid green line).

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Multivariate machine learning-based prediction models of freeway traffic flow under non-recurrent events, Alexandria Engineering Journal, Volume 65, 15 February 2023, Pages 151-162 at [Multivariate machine learning-based prediction models of freeway traffic flow under non-recurrent events - ScienceDirect]

Prediction - MLP model

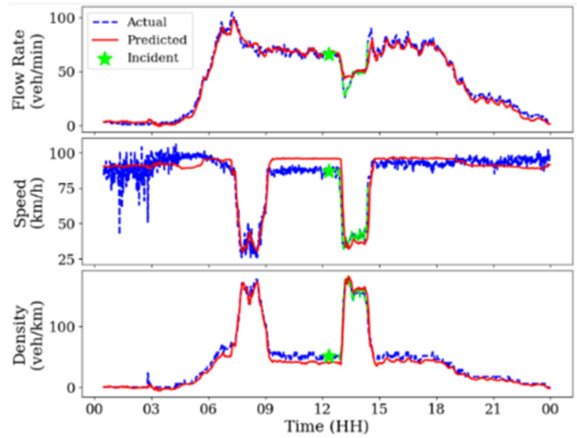

Figure: MLP predictions of traffic flow rate (veh/min/lane), speed (km/hr) and density (veh/km/lane) with a road incident (∗) and its impact on traffic parameters (a solid green line).

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Prediction - 1D CNN LSTMs model

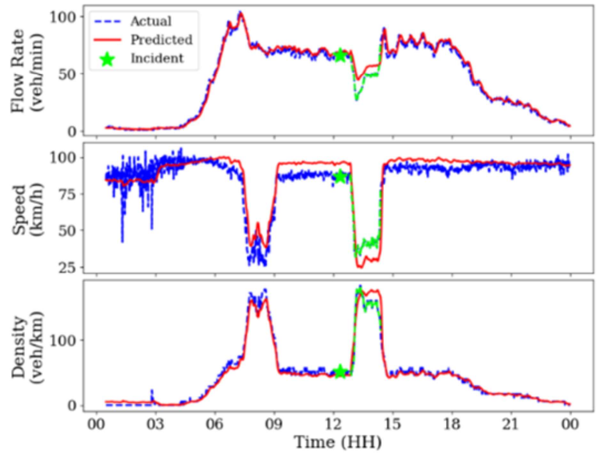

Figure: 1D-CNN LSTMs predictions of traffic flow rate (veh/min/lane), speed (km/hr) and density (veh/km/lane) with a road incident (∗) and its impact on traffic parameters (a solid green line).

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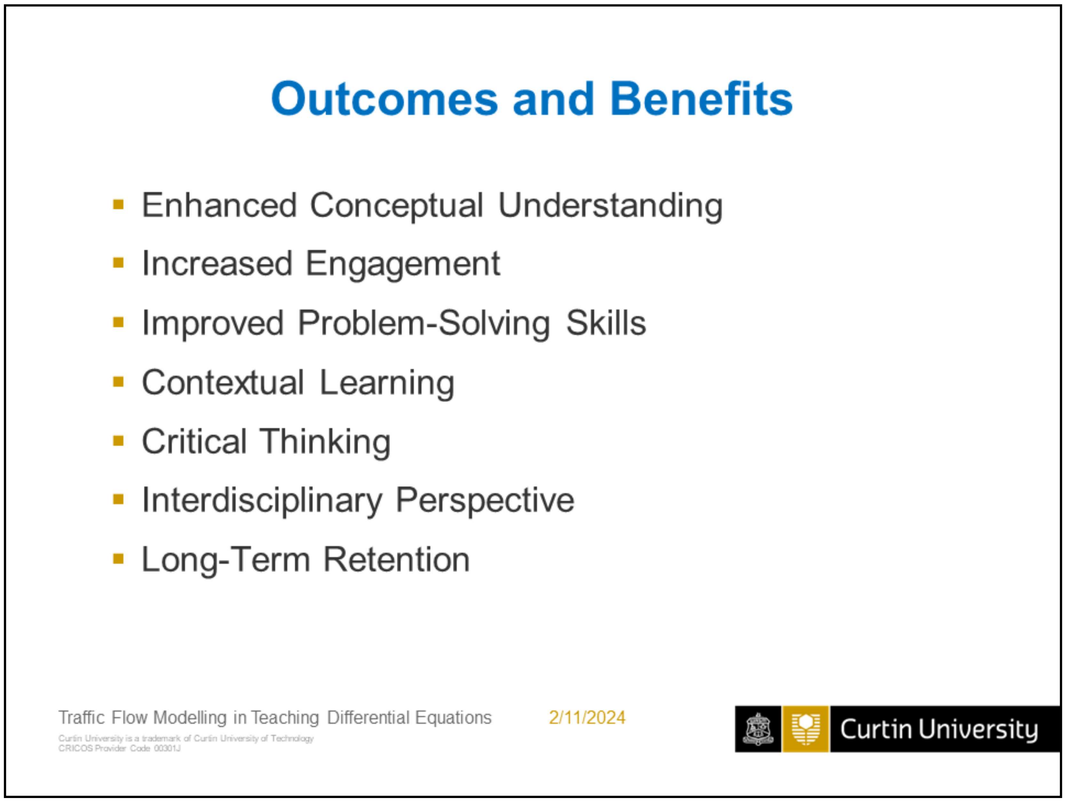


1.Enhanced Conceptual Understanding: Students gain a deeper grasp of differential equations as they see how these mathematical concepts directly relate to real-world traffic phenomena.

2.Increased Engagement: The practical application of traffic flow models captures students' interest and motivates them to explore mathematical theories more thoroughly.

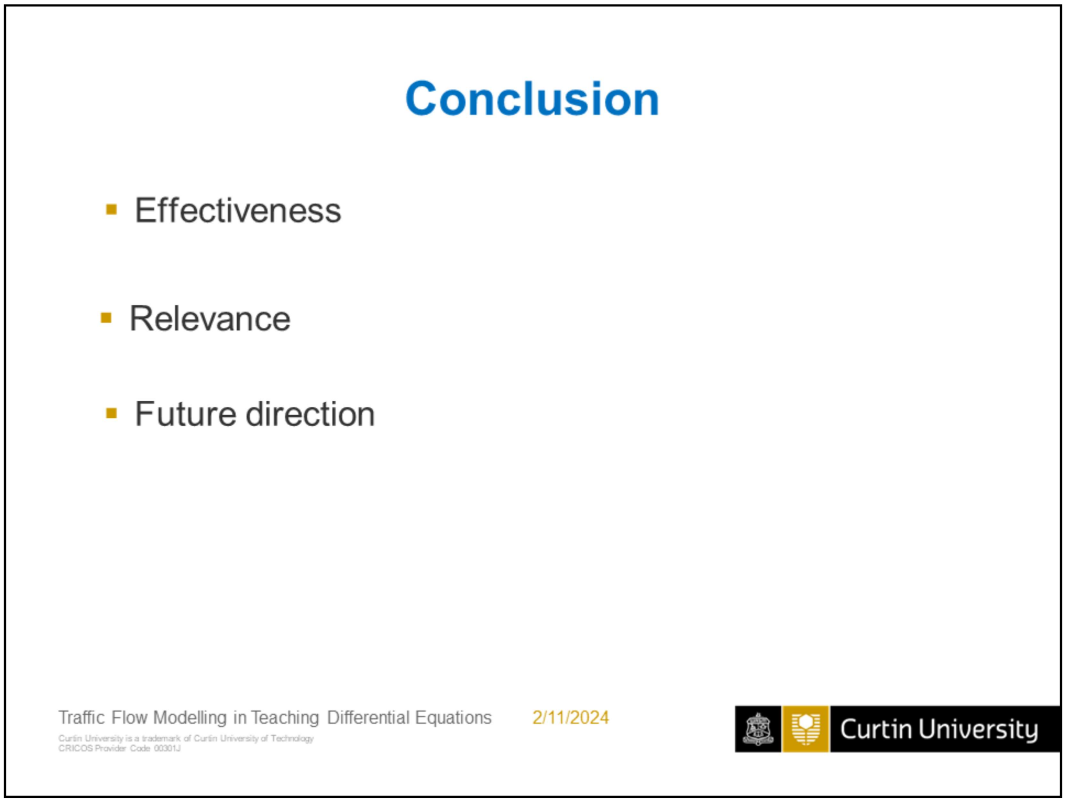
3.Improved Problem-Solving Skills: Integrating traffic flow modeling challenges students to apply their mathematical knowledge to real problems, fostering problem-solving skills that are valuable in various domains.

4.Contextual Learning: Students learn in context, which helps them appreciate the relevance of mathematical concepts in solving complex, real-world challenges.

5.Critical Thinking: Analyzing and adapting traffic flow models requires critical thinking, encouraging students to think critically and adapt mathematical theories to practical situations.

6.Interdisciplinary Perspective: Students develop an interdisciplinary perspective by understanding how mathematical modeling connects to fields such as transportation engineering, urban planning, and environmental science.

7.Long-Term Retention: Real-world applications are often more memorable, leading to better retention of mathematical principles for future studies and career applications.



1.Effectiveness of Real-World Applications: Using real-world applications, such as traffic flow modeling, in teaching complex mathematical theories has proven highly effective in several ways:

1. Enhances student engagement and motivation.
2. Improves conceptual understanding of abstract mathematical concepts.
3. Develops critical thinking and problem-solving skills.
4. Increases the practical relevance and applicability of mathematical theories.

2.Relevance of Traffic Flow Modeling: Traffic flow modeling, as a real-world application, holds particular significance in differential equations education:

1. Offers a tangible and relatable case study for students.
2. Illustrates the power and versatility of differential equations in solving practical problems.
3. Prepares students for careers in transportation engineering, urban planning, and related fields.
4. Demonstrates the interdisciplinary nature of mathematical modeling.

3.Future Directions: As we continue to evolve mathematics education, there is a growing need to integrate more real-world applications into the curriculum to better prepare students for the challenges of the modern world.

Questions and Discussion

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Thank the audience for their attention